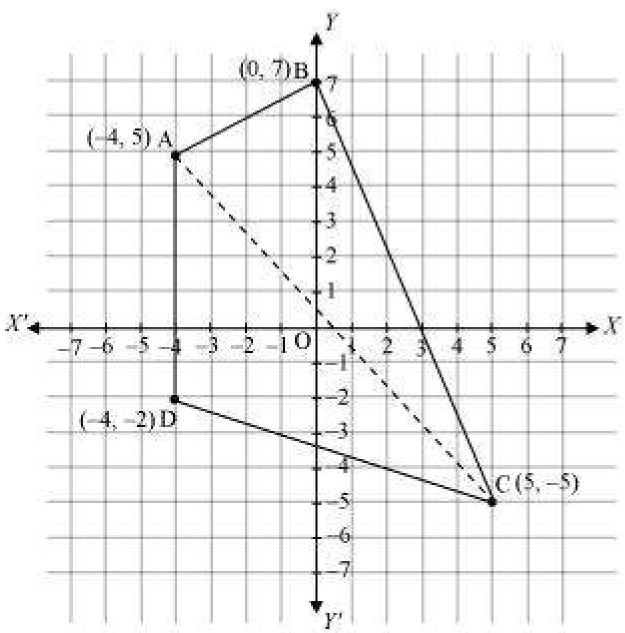
EXERCISE:-10.1

Question 1:

Draw a quadrilateral in the Cartesian plane, whose vertices are (–4, 5), (0, 7), (5, –5) and (–4, –2). Also, find its area.

Let ABCD be the given quadrilateral with vertices A (–4, 5), B (0, 7), C (5, –5), and D (–4, –2).

Then, by plotting A, B, C, and D on the Cartesian plane and joining AB, BC, CD, and DA, the given quadrilateral can be drawn as



To find the area of quadrilateral ABCD, we draw one diagonal, say AC.

Accordingly, area (ABCD) = area (ΔABC) + area (ΔACD)

We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Therefore, area of ΔABC

$= \frac{1}{2}\left|-4(7+5)+0(-5-5)+5(5-7)\right|$ unit^2

$= \frac{1}{2}\left|-4(12)+5(-2)\right|$ unit^2

$= \frac{1}{2}\left|-48-10\right|$ unit^2

$= \frac{1}{2}\left|-58\right|$ unit^2

$= \frac{1}{2}\times 58$ unit^2

$= 29$ unit^2

Area of ΔACD

$= \frac{1}{2}\left|-4(-5+2)+5(-2-5)+(-4)(5+5)\right|$ unit^2

$= \frac{1}{2}\left|-4(-3)+5(-7)-4(10)\right|$ unit^2

$= \frac{1}{2}\left|12-35-40\right|$ unit^2

$= \frac{1}{2}\left|-63\right|$ unit^2

$= \frac{63}{2}$ unit^2

Thus, area (ABCD) $= \left(29+\frac{63}{2}\right)$ unit^2 $= \frac{58+63}{2}$ unit^2 $= \frac{121}{2}$ unit^2

Question 2:

The base of an equilateral triangle with side 2*a* lies along they *y*-axis such that the mid point of the base is at the origin. Find vertices of the triangle.

Let ABC be the given equilateral triangle with side 2*a*.

Accordingly, AB = BC = CA = 2*a*

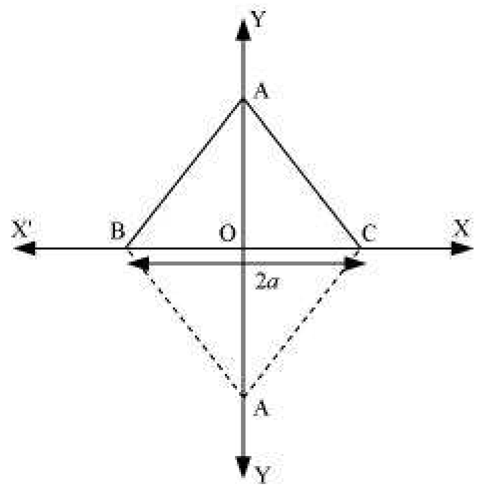
Assume that base BC lies along the *y*-axis such that the mid-point of BC is at the origin.

i.e., BO = OC = *a*, where O is the origin.

Now, it is clear that the coordinates of point C are (0, a), while the coordinates of point B are (0, $-a$).

It is known that the line joining a vertex of an equilateral triangle with the mid-point of its opposite side is perpendicular.

Hence, vertex A lies on the y-axis.



On applying Pythagoras theorem to ΔAOC, we obtain

$(AC)^2 = (OA)^2 + (OC)^2$

$\Rightarrow (2a)^2 = (OA)^2 + a^2$

$\Rightarrow 4a^2 - a^2 = (OA)^2$

$\Rightarrow (OA)^2 = 3a^2$

$\Rightarrow OA = \sqrt{3}a$

$\therefore$Coordinates of point A = $\left(\pm\sqrt{3}a, 0\right)$

Thus, the vertices of the given equilateral triangle are (0, a), (0, $-a$), and $\left(\sqrt{3}a,\ 0\right)$ or (0, a), (0, $-a$), and $\left(-\sqrt{3}a,\ 0\right)$.

Question 3:

Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when: (i) PQ is parallel to the *y*-axis, (ii) PQ is parallel to the *x*-axis.

The given points are $P(x_1, y_1)$ and $Q(x_2, y_2)$.

(i) When PQ is parallel to the *y*-axis, $x_1 = x_2$.

In this case, distance between P and Q $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(y_2 - y_1)^2}$$
$$= |y_2 - y_1|$$

(ii) When PQ is parallel to the *x*-axis, $y_1 = y_2$.

In this case, distance between P and Q $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(x_2 - x_1)^2}$$
$$= |x_2 - x_1|$$

Question 4:

Find a point on the *x*-axis, which is equidistant from the points (7, 6) and (3, 4).

Let $(a, 0)$ be the point on the *x* axis that is equidistant from the points (7, 6) and (3, 4).

Accordingly, $\sqrt{(7-a)^2 + (6-0)^2} = \sqrt{(3-a)^2 + (4-0)^2}$

$$\Rightarrow \sqrt{49 + a^2 - 14a + 36} = \sqrt{9 + a^2 - 6a + 16}$$
$$\Rightarrow \sqrt{a^2 - 14a + 85} = \sqrt{a^2 - 6a + 25}$$

On squaring both sides, we obtain

$a^2 - 14a + 85 = a^2 - 6a + 25$

$\Rightarrow -14a + 6a = 25 - 85$

$\Rightarrow -8a = -60$

$\Rightarrow a = \frac{60}{8} = \frac{15}{2}$

Thus, the required point on the x-axis is $\left(\frac{15}{2},\ 0\right)$.

Question 5:

Find the slope of a line, which passes through the origin, and the mid-point of

the line segment joining the points P (0, –4) and B (8, 0).

The coordinates of the mid-point of the line segment joining the points

P (0, –4) and B (8, 0) are $\left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4,\ -2)$

It is known that the slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2,y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$.

Therefore, the slope of the line passing through (0, 0) and (4, –2) is

$\frac{-2-0}{4-0} = \frac{-2}{4} = -\frac{1}{2}$.

Hence, the required slope of the line is $-\frac{1}{2}$.

Question 6:

Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right angled triangle.

The vertices of the given triangle are A (4, 4), B (3, 5), and C (–1, –1).

It is known that the slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2,y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$.

$\therefore$Slope of AB (m_1) $= \frac{5-4}{3-4} = -1$

Slope of BC (m_2) $= \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of CA (m_3) $= \frac{4+1}{4+1} = \frac{5}{5} = 1$

It is observed that $m_1 m_3 = -1$

This shows that line segments AB and CA are perpendicular to each other

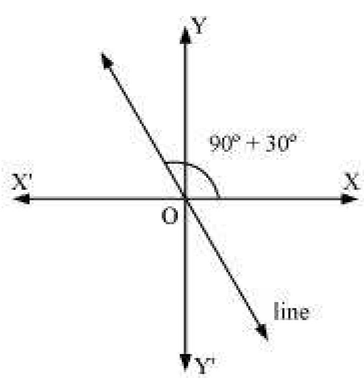
i.e., the given triangle is right-angled at A (4, 4).

Thus, the points (4, 4), (3, 5), and (–1, –1) are the vertices of a right-angled triangle.

Question 7:

Find the slope of the line, which makes an angle of 30° with the positive direction of y-axis measured anticlockwise.

If a line makes an angle of 30° with the positive direction of the y-axis measured anticlockwise, then the angle made by the line with the positive direction of the x-axis measured anticlockwise is 90° + 30° = 120°.



Thus, the slope of the given line is tan 120° = tan (180° – 60°) = –tan 60° $= -\sqrt{3}$

Question 8:

Find the value of *x* for which the points (*x*, –1), (2, 1) and (4, 5) are collinear.

If points A (*x*, –1), B (2, 1), and C (4, 5) are collinear, then

Slope of AB = Slope of BC

$$\Rightarrow \frac{1-(-1)}{2-x} = \frac{5-1}{4-2}$$
$$\Rightarrow \frac{1+1}{2-x} = \frac{4}{2}$$
$$\Rightarrow \frac{2}{2-x} = 2$$
$$\Rightarrow 2 = 4-2x$$
$$\Rightarrow 2x = 2$$
$$\Rightarrow x = 1$$

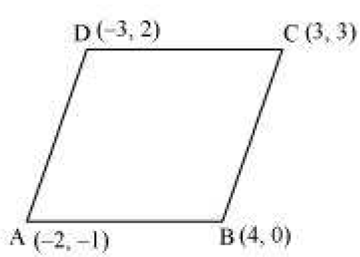
Thus, the required value of *x* is 1.

Question 9:

Without using distance formula, show that points (–2, –1), (4, 0), (3, 3) and

(–3, 2) are vertices of a parallelogram.

Let points (–2, –1), (4, 0), (3, 3), and (–3, 2) be respectively denoted by A, B, C, and D.



Slope of AB $= \frac{0+1}{4+2} = \frac{1}{6}$

Slope of CD = $\frac{2-3}{-3-3}=\frac{-1}{-6}=\frac{1}{6}$

⇒ Slope of AB = Slope of CD

⇒ AB and CD are parallel to each other.

Now, slope of BC = $\frac{3-0}{3-4}=\frac{3}{-1}=-3$

Slope of AD = $\frac{2+1}{-3+2}=\frac{3}{-1}=-3$

⇒ Slope of BC = Slope of AD

⇒ BC and AD are parallel to each other.

Therefore, both pairs of opposite sides of quadrilateral ABCD are parallel. Hence, ABCD is a parallelogram.

Thus, points (–2, –1), (4, 0), (3, 3), and (–3, 2) are the vertices of a parallelogram.

Question 10:

Find the angle between the x-axis and the line joining the points (3, –1) and (4, –2).

The slope of the line joining the points (3, –1) and (4, –2) is $m=\frac{-2-(-1)}{4-3}=-2+1=-1$

Now, the inclination (θ) of the line joining the points (3, –1) and (4, – 2) is given by

$\tan\theta = -1$

⇒ $\theta = (90° + 45°) = 135°$

Thus, the angle between the x-axis and the line joining the points (3, –1) and (4, –2) is 135°.

Question 11:

The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of he lines.

Let m_1 and m be the slopes of the two given lines such that $m_1 = 2m$.

We know that if θisthe angle between the lines l_1 and l_2 with slopes m_1 and m_2, then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$.

It is given that the tangent of the angle between the two lines is $\frac{1}{3}$.

$$\therefore \frac{1}{3} = \left|\frac{m - 2m}{1 + (2m)\cdot m}\right|$$
$$\Rightarrow \frac{1}{3} = \left|\frac{-m}{1 + 2m^2}\right|$$
$$\Rightarrow \frac{1}{3} = \frac{-m}{1 + 2m^2} \text{ or } \frac{1}{3} = -\left(\frac{-m}{1 + 2m^2}\right) = \frac{m}{1 + 2m^2}$$

Case I

$$\Rightarrow \frac{1}{3} = \frac{-m}{1 + 2m^2}$$
$$\Rightarrow 1 + 2m^2 = -3m$$
$$\Rightarrow 2m^2 + 3m + 1 = 0$$
$$\Rightarrow 2m^2 + 2m + m + 1 = 0$$
$$\Rightarrow 2m(m+1) + 1(m+1) = 0$$
$$\Rightarrow (m+1)(2m+1) = 0$$
$$\Rightarrow m = -1 \text{ or } m = -\frac{1}{2}$$

If $m = -1$, then the slopes of the lines are -1 and -2.

If $m = -\frac{1}{2}$, then the slopes of the lines are $-\frac{1}{2}$ and -1.

Case II

$$\frac{1}{3}=\frac{m}{1+2m^2}$$
$$\Rightarrow 2m^2+1=3m$$
$$\Rightarrow 2m^2-3m+1=0$$
$$\Rightarrow 2m^2-2m-m+1=0$$
$$\Rightarrow 2m(m-1)-1(m-1)=0$$
$$\Rightarrow (m-1)(2m-1)=0$$
$$\Rightarrow m=1 \text{ or } m=\frac{1}{2}$$

If m = 1, then the slopes of the lines are 1 and 2.

If $m = \frac{1}{2}$, then the slopes of the lines are $\frac{1}{2}$ and 1.

Hence, the slopes of the lines are –1 and –2 or $-\frac{1}{2}$ and –1 or 1 and 2 or $\frac{1}{2}$ and 1.

Question 12:

A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k-y_1=m(h-x_1)$.

The slope of the line passing through (x_1, y_1) and (h, k) is $\frac{k-y_1}{h-x_1}$.

It is given that the slope of the line is m.

$$\therefore \frac{k-y_1}{h-x_1}=m$$
$$\Rightarrow k-y_1=m(h-x_1)$$

Hence, $k-y_1=m(h-x_1)$

Question 13:

If three point $(h, 0)$, (a, b) and $(0, k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$.

If the points A $(h, 0)$, B (a, b), and C $(0, k)$ lie on a line, then

Slope of AB = Slope of BC

$$\frac{b-0}{a-h}=\frac{k-b}{0-a}$$
$$\Rightarrow \frac{b}{a-h}=\frac{k-b}{-a}$$
$$\Rightarrow -ab=(k-b)(a-h)$$
$$\Rightarrow -ab=ka-kh-ab+bh$$
$$\Rightarrow ka+bh=kh$$

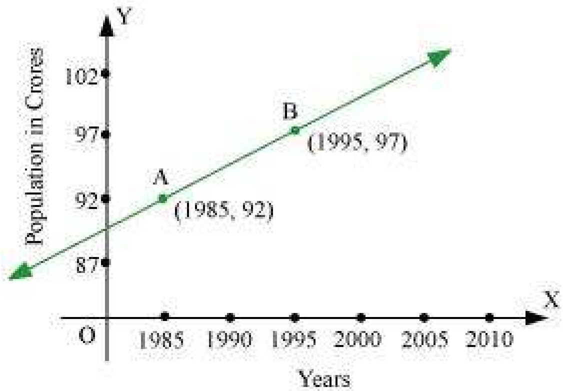
On dividing both sides by kh, we obtain

$$\frac{ka}{kh}+\frac{bh}{kh}=\frac{kh}{kh}$$
$$\Rightarrow \frac{a}{h}+\frac{b}{k}=1$$

Hence, $\frac{a}{h}+\frac{b}{k}=1$

Question 14:

Consider the given population and year graph. Find the slope of the line AB and using it, find what will be the population in the year 2010?



Since line AB passes through points A (1985, 92) and B (1995, 97), its slope is $\frac{97-92}{1995-1985}=\frac{5}{10}=\frac{1}{2}$

Let y be the population in the year 2010. Then, according to the given graph, line AB must pass through point C (2010, y).

$\therefore$Slope of AB = Slope of BC

$$\Rightarrow \frac{1}{2}=\frac{y-97}{2010-1995}$$
$$\Rightarrow \frac{1}{2}=\frac{y-97}{15}$$
$$\Rightarrow \frac{15}{2}=y-97$$
$$\Rightarrow y-97=7.5$$
$$\Rightarrow y=7.5+97=104.5$$

Thus, the slope of line AB is $\frac{1}{2}$, while in the year 2010, the population will be 104.5 crores.

EXERCISE:-10.2

Question 1:

Write the equations for the x and y-axes.

The y-coordinate of every point on the x-axis is 0.

Therefore, the equation of the x-axis is $y = 0$.

The x-coordinate of every point on the y-axis is 0.

Therefore, the equation of the y-axis is $y = 0$.

Question 2:

Find the equation of the line which passes through the point (–4, 3) with slope $\frac{1}{2}$.

We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is $(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point (–4, 3), whose slope is $\frac{1}{2}$, is

$$(y-3) = \frac{1}{2}(x+4)$$
$$2(y-3) = x+4$$
$$2y-6 = x+4$$
$$\text{i.e., } x-2y+10 = 0$$

Question 3:

Find the equation of the line which passes though (0, 0) with slope m.

We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is $(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point (0, 0), whose slope is m,is

$(y - 0) = m(x - 0)$

i.e., $y = mx$

Question 4:

Find the equation of the line which passes though $(2,\ 2\sqrt{3})$ and is inclined with the *x*-axis at an angle of 75°.

The slope of the line that inclines with the *x*-axis at an angle of 75° is

$m = \tan 75°$

$$\Rightarrow m = \tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

We know that the equation of the line passing through point (x_0, y_0), whose slope is *m*, is $(y - y_0) = m(x - x_0)$.

Thus, if a line passes though $(2,\ 2\sqrt{3})$ and inclines with the *x*-axis at an angle of 75°, then the equation of the line is given as

$$\left(y - 2\sqrt{3}\right) = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$
$$\left(y - 2\sqrt{3}\right)\left(\sqrt{3}-1\right) = \left(\sqrt{3}+1\right)(x-2)$$
$$y\left(\sqrt{3}-1\right) - 2\sqrt{3}\left(\sqrt{3}-1\right) = x\left(\sqrt{3}+1\right) - 2\left(\sqrt{3}+1\right)$$
$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}$$
$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\sqrt{3} - 4$$
$$\text{i.e., } \left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\left(\sqrt{3}-1\right)$$

Question 5:

Find the equation of the line which intersects the *x*-axis at a distance of 3 units to the left of origin with slope –2.

It is known that if a line with slope *m* makes *x*-intercept *d*, then the equation of the line is given as

$y = m(x - d)$

For the line intersecting the x-axis at a distance of 3 units to the left of the origin, $d = -3$.

The slope of the line is given as $m = -2$

Thus, the required equation of the given line is

$y = -2\,[x - (-3)]$

$y = -2x - 6$

i.e., $2x + y + 6 = 0$

Question 6:

Find the equation of the line which intersects the y-axis at a distance of 2 units above the origin and makes an angle of 30° with the positive direction of the x-axis.

It is known that if a line with slope m makes y-intercept c, then the equation of the line is given as

$y = mx + c$

Here, $c = 2$ and $m = \tan 30° = \frac{1}{\sqrt{3}}$.

Thus, the required equation of the given line is

$$y = \frac{1}{\sqrt{3}}x + 2$$
$$y = \frac{x + 2\sqrt{3}}{\sqrt{3}}$$
$$\sqrt{3}y = x + 2\sqrt{3}$$
$$\text{i.e., } x - \sqrt{3}y + 2\sqrt{3} = 0$$

Question 7:

Find the equation of the line which passes through the points (–1, 1) and (2, –4).

It is known that the equation of the line passing through points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Therefore, the equation of the line passing through the points (–1, 1) and

(2, –4) is

$$(y-1) = \frac{-4-1}{2+1}(x+1)$$
$$(y-1) = \frac{-5}{3}(x+1)$$
$$3(y-1) = -5(x+1)$$
$$3y - 3 = -5x - 5$$
$$\text{i.e., } 5x + 3y + 2 = 0$$

Question 8:

Find the equation of the line which is at a perpendicular distance of 5 units from the origin and the angle made by the perpendicular with the positive x-axis is 30°

If p is the length of the normal from the origin to a line and ω is the angle made by the normal with the positive direction of the x-axis, then the equation of the line is given by $x\cos \omega + y \sin\omega = p$.

Here, $p = 5$ units and $\omega = 30°$

Thus, the required equation of the given line is

$x \cos 30° + y \sin 30° = 5$

$$x\frac{\sqrt{3}}{2} + y\cdot\frac{1}{2} = 5$$
$$\text{i.e., } \sqrt{3}x + y = 10$$

Question 9:

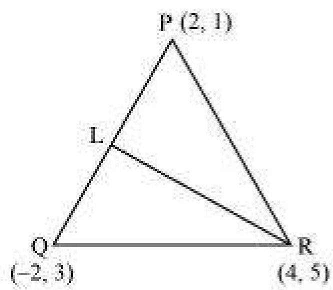
The vertices of ΔPQR are P (2, 1), Q (–2, 3) and R (4, 5). Find equation of the median through the vertex R.

It is given that the vertices of ΔPQR are P (2, 1), Q (–2, 3), and R (4, 5).

Let RL be the median through vertex R.

Accordingly, L is the mid-point of PQ.

By mid-point formula, the coordinates of point L are given by $\left(\frac{2-2}{2}, \frac{1+3}{2}\right) = (0, 2)$



It is known that the equation of the line passing through points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

Therefore, the equation of RL can be determined by substituting $(x_1, y_1) = (4, 5)$ and $(x_2, y_2) = (0, 2)$.

Hence, $y - 5 = \frac{2-5}{0-4}(x-4)$

$$\Rightarrow y - 5 = \frac{-3}{-4}(x-4)$$
$$\Rightarrow 4(y-5) = 3(x-4)$$
$$\Rightarrow 4y - 20 = 3x - 12$$
$$\Rightarrow 3x - 4y + 8 = 0$$

Thus, the required equation of the median through vertex R is $3x - 4y + 8 = 0$.

Question 10:

Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

The slope of the line joining the points (2, 5) and (–3, 6) is $m=\frac{6-5}{-3-2}=\frac{1}{-5}$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line perpendicular to the line through the points (2, 5) and (–3, 6)

$$=-\frac{1}{m}=-\frac{1}{\left(\frac{-1}{5}\right)}=5$$

Now, the equation of the line passing through point (–3, 5), whose slope is 5, is

$$\begin{aligned}&(y-5)=5(x+3)\\&y-5=5x+15\\&\text{i.e., } 5x-y+20=0\end{aligned}$$

Question 11:

A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1:*n*. Find the equation of the line.

According to the section formula, the coordinates of the point that divides the line segment joining the points (1, 0) and (2, 3) in the ratio 1: *n* is given by

$$\left(\frac{n(1)+1(2)}{1+n},\frac{n(0)+1(3)}{1+n}\right)=\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$$

The slope of the line joining the points (1, 0) and (2, 3) is

$$m=\frac{3-0}{2-1}=3$$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points (1, 0) and (2, 3) $= -\frac{1}{m} = -\frac{1}{3}$

Now, the equation of the line passing through $\left(\frac{n+2}{n+1}, \frac{3}{n+1}\right)$ and whose slope is $-\frac{1}{3}$ is given by

$$\left(y - \frac{3}{n+1}\right) = \frac{-1}{3}\left(x - \frac{n+2}{n+1}\right)$$
$$\Rightarrow 3\left[(n+1)y - 3\right] = -\left[x(n+1) - (n+2)\right]$$
$$\Rightarrow 3(n+1)y - 9 = -(n+1)x + n + 2$$
$$\Rightarrow (1+n)x + 3(1+n)y = n + 11$$

Question 12:

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

The equation of a line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad \text{... (i)}$$

Here, *a* and *b* are the intercepts on *x* and *y* axes respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that $a = b$.

Accordingly, equation (i) reduces to

$$\frac{x}{a} + \frac{y}{a} = 1$$
$$\Rightarrow x + y = a \qquad \text{... (ii)}$$

Since the given line passes through point (2, 3), equation (ii) reduces to

$2 + 3 = a \Rightarrow a = 5$

On substituting the value of a in equation (ii), we obtain

$x + y = 5$, which is the required equation of the line

Question 13:

Find equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \text{... (i)}$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that$a + b = 9 \Rightarrow b = 9 - a$ … (ii)

From equations (i) and (ii), we obtain

$$\frac{x}{a}+\frac{y}{9-a}=1 \qquad \text{...(iii)}$$

It is given that the line passes through point (2, 2). Therefore, equation (iii) reduces to

$$\frac{2}{a}+\frac{2}{9-a}=1$$
$$\Rightarrow 2\left(\frac{1}{a}+\frac{1}{9-a}\right)=1$$
$$\Rightarrow 2\left(\frac{9-a+a}{a(9-a)}\right)=1$$
$$\Rightarrow \frac{18}{9a-a^2}=1$$
$$\Rightarrow 18=9a-a^2$$
$$\Rightarrow a^2-9a+18=0$$
$$\Rightarrow a^2-6a-3a+18=0$$
$$\Rightarrow a(a-6)-3(a-6)=0$$
$$\Rightarrow (a-6)(a-3)=0$$
$$\Rightarrow a=6 \text{ or } a=3$$

If a = 6 and b = 9 – 6 = 3, then the equation of the line is

$$\frac{x}{6}+\frac{y}{3}=1 \Rightarrow x+2y-6=0$$

If a = 3 and b = 9 – 3 = 6, then the equation of the line is

$$\frac{x}{3}+\frac{y}{6}=1 \Rightarrow 2x+y-6=0$$

Question 14:

Find equation of the line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

The slope of the line making an angle $\frac{2\pi}{3}$ with the positive x-axis is $m=\tan\left(\frac{2\pi}{3}\right)=-\sqrt{3}$

Now, the equation of the line passing through point (0, 2) and having a slope $-\sqrt{3}$ is $(y-2)=-\sqrt{3}(x-0)$.

$$y-2=-\sqrt{3}x$$
i.e., $\sqrt{3}x+y-2=0$

The slope of line parallel to line $\sqrt{3}x+y-2=0$ is $-\sqrt{3}$.

It is given that the line parallel to line $\sqrt{3}x+y-2=0$ crosses the *y*-axis 2 units below the origin i.e., it passes through point (0, –2).

Hence, the equation of the line passing through point (0, –2) and having a slope $-\sqrt{3}$ is

$$y-(-2)=-\sqrt{3}(x-0)$$
$$y+2=-\sqrt{3}x$$
$$\sqrt{3}x+y+2=0$$

Question 15:

The perpendicular from the origin to a line meets it at the point (– 2, 9), find the equation of the line.

The slope of the line joining the origin (0, 0) and point (–2, 9) is $m_1=\frac{9-0}{-2-0}=-\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and point (–2, 9) is

$$m_2=-\frac{1}{m_1}=-\frac{1}{\left(-\frac{9}{2}\right)}=\frac{2}{9}$$

Now, the equation of the line passing through point (–2, 9) and having a slope m_2 is

$$(y-9)=\frac{2}{9}(x+2)$$
$$9y-81=2x+4$$
i.e., $2x-9y+85=0$

Question 16:

The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L = 125.134 when C = 110, express L in terms of C.

It is given that when C = 20, the value of L is 124.942, whereas when C = 110, the value of L is 125.134.

Accordingly, points (20, 124.942) and (110, 125.134) satisfy the linear relation between L and C.

Now, assuming C along the *x*-axis and L along the *y*-axis, we have two points i.e., (20, 124.942) and (110, 125.134) in the XY plane.

Therefore, the linear relation between L and C is the equation of the line passing through points (20, 124.942) and (110, 125.134).

$$(L - 124.942) = \frac{125.134 - 124.942}{110 - 20}(C - 20)$$

$$L - 124.942 = \frac{0.192}{90}(C - 20)$$

i.e., $L = \frac{0.192}{90}(C - 20) + 124.942$, which is the required linear relation

Question 17:

The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

The relationship between selling price and demand is linear.

Assuming selling price per litre along the *x*-axis and demand along the *y*-axis, we have two points i.e., (14, 980) and (16, 1220) in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the linear relationship between selling price per litre and demand is the equation of the line passing through points (14, 980) and (16, 1220).

$$y-980=\frac{1220-980}{16-14}(x-14)$$

$$y-980=\frac{240}{2}(x-14)$$
$$y-980=120(x-14)$$
i.e., $y=120(x-14)+980$

When x = Rs 17/litre,

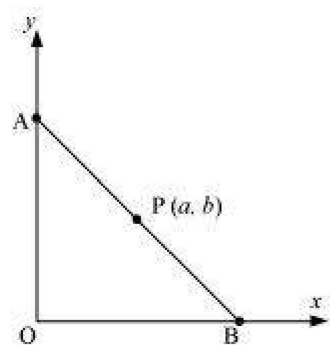
$$y=120(17-14)+980$$
$$\Rightarrow y=120\times 3+980=360+980=1340$$

Thus, the owner of the milk store could sell 1340 litres of milk weekly at Rs 17/litre.

Question 18:

P (*a, b*) is the mid-point of a line segment between axes. Show that equation of the line is $\frac{x}{a}+\frac{y}{b}=2$

Let AB be the line segment between the axes and let P (*a*, *b*) be its mid-point.



Let the coordinates of A and B be (0, *y*) and (*x*, 0) respectively.

Since P (*a*, *b*) is the mid-point of AB,

$$\left(\frac{0+x}{2},\frac{y+0}{2}\right)=(a,b)$$
$$\Rightarrow\left(\frac{x}{2},\frac{y}{2}\right)=(a,b)$$
$$\Rightarrow\frac{x}{2}=a \text{ and } \frac{y}{2}=b$$
$$\therefore x=2a \text{ and } y=2b$$

Thus, the respective coordinates of A and B are (0, 2*b*) and (2*a*, 0).

The equation of the line passing through points (0, 2*b*) and (2*a*, 0) is

$$(y-2b)=\frac{(0-2b)}{(2a-0)}(x-0)$$
$$y-2b=\frac{-2b}{2a}(x)$$
$$a(y-2b)=-bx$$
$$ay-2ab=-bx$$
$$\text{i.e., } bx+ay=2ab$$

On dividing both sides by *ab*, we obtain

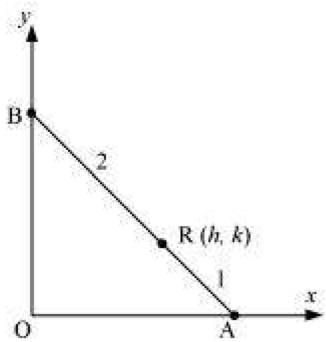
$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{2ab}{ab}$$
$$\Rightarrow\frac{x}{a}+\frac{y}{b}=2$$

Thus,the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$.

Question 19:

Point R (*h, k*) divides a line segment between the axes in the ratio 1:2. Find equation of the line.

Let AB be the line segment between the axes such that point R (*h*, *k*) divides AB in the ratio 1: 2.



Let the respective coordinates of A and B be (x, 0) and (0, y).

Since point R (h, k) divides AB in the ratio 1: 2, according to the section formula,

$$(h,k)=\left(\frac{1\times 0+2\times x}{1+2},\frac{1\times y+2\times 0}{1+2}\right)$$
$$\Rightarrow (h,k)=\left(\frac{2x}{3},\frac{y}{3}\right)$$
$$\Rightarrow h=\frac{2x}{3} \text{ and } k=\frac{y}{3}$$
$$\Rightarrow x=\frac{3h}{2} \text{ and } y=3k$$

Therefore, the respective coordinates of A and B are $\left(\frac{3h}{2},0\right)$ and (0, 3k).

Now, the equation of line AB passing through points $\left(\frac{3h}{2},0\right)$ and

(0, 3k) is

$$(y-0)=\frac{3k-0}{0-\frac{3h}{2}}\left(x-\frac{3h}{2}\right)$$
$$y=-\frac{2k}{h}\left(x-\frac{3h}{2}\right)$$
$$hy=-2kx+3hk$$
$$\text{i.e., } 2kx+hy=3hk$$

Thus,the required equation of the line is $2kx + hy = 3hk$.

Question 20:

By using the concept of equation of a line, prove that the three points (3, 0),

(–2, –2) and (8, 2) are collinear.

In order to show that points (3, 0), (–2, –2), and (8, 2) are collinear, it suffices to show that the line passing through points (3, 0) and (–2, –2) also passes through point (8, 2).

The equation of the line passing through points (3, 0) and (–2, –2) is

$$(y-0)=\frac{(-2-0)}{(-2-3)}(x-3)$$
$$y=\frac{-2}{-5}(x-3)$$
$$5y=2x-6$$
$$\text{i.e., } 2x-5y=6$$

It is observed that at $x = 8$ and $y = 2$,

L.H.S. = 2 × 8 – 5 × 2 = 16 – 10 = 6 = R.H.S.

Therefore, the line passing through points (3, 0) and (–2, –2) also passes through point (8, 2). Hence, points (3, 0), (–2, –2), and (8, 2) are collinear.

EXERCISE:-10.3

Question 1:

Reduce the following equations into slope-intercept form and find their slopes and the y-intercepts.

(i) $x + 7y = 0$ (ii) $6x + 3y – 5 = 0$ (iii) $y = 0$

(i) The given equation is $x + 7y = 0$.

It can be written as

$$y = -\frac{1}{7}x + 0 \qquad ...(1)$$

This equation is of the form $y = mx + c$, where $m = -\frac{1}{7}$ and $c = 0$.

Therefore, equation (1) is in the slope-intercept form, where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x + 3y - 5 = 0$.

It can be written as

$$y = \frac{1}{3}(-6x + 5)$$
$$y = -2x + \frac{5}{3} \qquad ...(2)$$

This equation is of the form $y = mx + c$, where $m = -2$ and $c = \frac{5}{3}$.

Therefore, equation (2) is in the slope-intercept form, where the slope and the y-intercept are–2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y = 0$.

It can be written as

$y = 0.x + 0$ … (3)

This equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$.

Therefore, equation (3) is in the slope-intercept form, where the slope and the y-intercept are 0 and 0 respectively.

Question 2:

Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$ (ii) $4x - 3y = 6$ (iii) $3y + 2 = 0$.

(i) The given equation is $3x + 2y - 12 = 0$.

It can be written as

$$3x + 2y = 12$$
$$\frac{3x}{12} + \frac{2y}{12} = 1$$

i.e., $\frac{x}{4} + \frac{y}{6} = 1$ ...(1)

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 4$ and $b = 6$.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

(ii) The given equation is $4x - 3y = 6$.

It can be written as

$$\frac{4x}{6} - \frac{3y}{6} = 1$$
$$\frac{2x}{3} - \frac{y}{2} = 1$$

i.e., $\frac{x}{\left(\frac{3}{2}\right)} + \frac{y}{(-2)} = 1$ ...(2)

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = \frac{3}{2}$ and $b = -2$.

Therefore, equation (2) is in the intercept form, where the intercepts on the *x* and *y* axes are $\frac{3}{2}$ and –2 respectively.

(iii) The given equation is $3y + 2 = 0$.

It can be written as

$$3y = -2$$

$$\text{i.e., } \frac{y}{\left(-\frac{2}{3}\right)} = 1 \qquad ...(3)$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 0$ and $b = -\frac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercept on the *y*-axis is $-\frac{2}{3}$ and it has no intercept on the *x*-axis.

Question 3:

Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive *x*-axis.

(i) $x - \sqrt{3}y + 8 = 0$ (ii) $y – 2 = 0$ (iii) $x – y = 4$

(i) The given equation is $x - \sqrt{3}y + 8 = 0$.

It can be reduced as:

$$x - \sqrt{3}y = -8$$
$$\Rightarrow -x + \sqrt{3}y = 8$$

On dividing both sides by $\sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{4} = 2$, we obtain

$$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$$
$$\Rightarrow\left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$$
$$\Rightarrow x\cos 120^\circ+y\sin 120^\circ=4 \qquad ...(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 120°$ and $p = 4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120°.

(ii) The given equation is $y - 2 = 0$.

It can be reduced as $0.x + 1.y = 2$

On dividing both sides by $\sqrt{0^2+1^2}=1$, we obtain $0.x + 1.y = 2$

$\Rightarrow x \cos 90° + y \sin 90° = 2$ … (1)

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 90°$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive x-axis is 90°.

(iii) The given equation is $x - y = 4$.

It can be reduced as $1.x + (-1)\,y = 4$

On dividing both sides by $\sqrt{1^2+(-1)^2}=\sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x+\left(-\frac{1}{\sqrt{2}}\right)y=\frac{4}{\sqrt{2}}$$

$$\Rightarrow x\cos\left(2\pi-\frac{\pi}{4}\right)+y\sin\left(2\pi-\frac{\pi}{4}\right)=2\sqrt{2}$$

$$\Rightarrow\ x\cos 315°+y\sin 315°=2\sqrt{2} \qquad ...(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 315°$ and $p=2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x-axis is 315°.

Question 4:

Find the distance of the point (–1, 1) from the line $12(x + 6) = 5(y – 2)$.

The given equation of the line is $12(x + 6) = 5(y – 2)$.

$\Rightarrow 12x + 72 = 5y – 10$

$\Rightarrow 12x – 5y + 82 = 0$ … (1)

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 12$, $B= –5$, and $C = 82$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

The given point is $(x_1, y_1) = (–1, 1)$.

Therefore, the distance of point (–1, 1) from the given line

$$=\frac{|12(-1)+(-5)(1)+82|}{\sqrt{(12)^2+(-5)^2}}\text{ units}=\frac{|-12-5+82|}{\sqrt{169}}\text{ units}=\frac{|65|}{13}\text{ units}=5\text{ units}$$

Question 5:

Find the points on the *x*-axis, whose distances from the line $\frac{x}{3}+\frac{y}{4}=1$ are 4 units.

The given equation of line is

$$\frac{x}{3}+\frac{y}{4}=1$$
$$\text{or, } 4x+3y-12=0 \qquad \ldots(1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the *x*-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

Therefore,

$$4=\frac{|4a+3\times 0-12|}{\sqrt{4^2+3^2}}$$
$$\Rightarrow 4=\frac{|4a-12|}{5}$$
$$\Rightarrow |4a-12|=20$$
$$\Rightarrow \pm(4a-12)=20$$
$$\Rightarrow (4a-12)=20 \text{ or } -(4a-12)=20$$
$$\Rightarrow 4a=20+12 \text{ or } 4a=-20+12$$
$$\Rightarrow a=8 \text{ or } -2$$

Thus, the required points on the *x*-axis are (–2, 0) and (8, 0).

Question 6:

Find the distance between parallel lines

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

It is known that the distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.

(i) The given parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Here, $A = 15$, $B = 8$, $C_1 = -34$, and $C_2 = 31$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} \text{ units} = \frac{|-65|}{17} \text{ units} = \frac{65}{17} \text{ units}$$

(ii) The given parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$.

$lx + ly + p = 0$ and $lx + ly - r = 0$

Here, $A = l$, $B = l$, $C_1 = p$, and $C_2 = -r$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|p + r|}{\sqrt{l^2 + l^2}} \text{ units} = \frac{|p + r|}{\sqrt{2l^2}} \text{ units} = \frac{|p + r|}{l\sqrt{2}} \text{ units} = \frac{1}{\sqrt{2}}\left|\frac{p + r}{l}\right| \text{ units}$$

Question 7:

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

The equation of the given line is

$3x-4y+2=0$

or $y=\frac{3x}{4}+\frac{2}{4}$

or $y=\frac{3}{4}x+\frac{1}{2}$, which is of the form $y = mx + c$

∴ Slope of the given line $=\frac{3}{4}$

It is known that parallel lines have the same slope.

∴ Slope of the other line = $m=\frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the point (–2, 3) is

$(y-3)=\frac{3}{4}\{x-(-2)\}$

$4y-12=3x+6$

i.e., $3x-4y+18=0$

Question 8:

Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

The given equation of line is $x-7y+5=0$.

Or, $y=\frac{1}{7}x+\frac{5}{7}$, which is of the form $y = mx + c$

∴Slope of the given line $=\frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is $m=-\frac{1}{\left(\frac{1}{7}\right)}=-7$

The equation of the line with slope –7 and x-intercept 3 is given by

$y = m(x - d)$

$\Rightarrow y = -7(x - 3)$

$\Rightarrow y = -7x + 21$

$\Rightarrow 7x + y = 21$

Question 9:

Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

The given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

$y = -\sqrt{3}x + 1$ ...(1) and $y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}$ ...(2)

The slope of line (1) is $m_1 = -\sqrt{3}$, while the slope of line (2) is $m_2 = -\frac{1}{\sqrt{3}}$.

The acute angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$$\tan\theta = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right|$$
$$\tan\theta = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2 \times \sqrt{3}}\right|$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$
$$\theta = 30^\circ$$

Thus, the angle between the given lines is either 30° or 180° – 30° = 150°.

Question 10:

The line through the points (h, 3) and (4, 1) intersects the line $7x - 9y - 19 = 0$. at right angle. Find the value of h.

The slope of the line passing through points (h, 3) and (4, 1) is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of line $7x - 9y - 19 = 0$ or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$.

It is given that the two lines are perpendicular.

$$\therefore m_1 \times m_2 = -1$$
$$\Rightarrow \left(\frac{-2}{4-h}\right) \times \left(\frac{7}{9}\right) = -1$$
$$\Rightarrow \frac{-14}{36-9h} = -1$$
$$\Rightarrow 14 = 36 - 9h$$
$$\Rightarrow 9h = 36 - 14$$
$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of h is $\frac{22}{9}$.

Question 11:

Prove that the line through the point (x_1, y_1) and parallel to the line Ax + By + C = 0 is A ($x - x_1$) +B ($y - y_1$) = 0.

The slope of line Ax + By + C = 0 or $y = \left(\frac{-A}{B}\right)x + \left(\frac{-C}{B}\right)$ is $m = -\frac{A}{B}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line = $m = -\frac{A}{B}$

The equation of the line passing through point (x_1, y_1) and having a slope $m = -\frac{A}{B}$ is

$$y - y_1 = m(x - x_1)$$
$$y - y_1 = -\frac{A}{B}(x - x_1)$$
$$B(y - y_1) = -A(x - x_1)$$
$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point (x_1, y_1) and parallel to line $Ax + By + C = 0$ is

$A(x - x_1) + B(y - y_1) = 0$

Question 12:

Two lines passing through the point (2, 3) intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2.

The angle between the two lines is 60°.

$$\therefore \tan 60° = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$$\Rightarrow \sqrt{3} = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$
$$\Rightarrow \sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$
$$\Rightarrow \sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2}\right)$$
$$\Rightarrow \sqrt{3}(1 + 2m_2) = 2 - m_2 \text{ or } \sqrt{3}(1 + 2m_2) = -(2 - m_2)$$
$$\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$
$$\Rightarrow \sqrt{3} + \left(2\sqrt{3} + 1\right)m_2 = 2 \text{ or } \sqrt{3} + \left(2\sqrt{3} - 1\right)m_2 = -2$$
$$\Rightarrow m_2 = \frac{2 - \sqrt{3}}{\left(2\sqrt{3} + 1\right)} \text{ or } m_2 = \frac{-\left(2 + \sqrt{3}\right)}{\left(2\sqrt{3} - 1\right)}$$

Case I : $m_2 = \left(\dfrac{2-\sqrt{3}}{2\sqrt{3}+1}\right)$

The equation of the line passing through point (2, 3) and having a slope of $\dfrac{\left(2-\sqrt{3}\right)}{\left(2\sqrt{3}+1\right)}$ is

$$(y-3) = \frac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$$
$$\left(2\sqrt{3}+1\right)y - 3\left(2\sqrt{3}+1\right) = \left(2-\sqrt{3}\right)x - 2\left(2-\sqrt{3}\right)$$
$$\left(\sqrt{3}-2\right)x + \left(2\sqrt{3}+1\right)y = -4+2\sqrt{3}+6\sqrt{3}+3$$
$$\left(\sqrt{3}-2\right)x + \left(2\sqrt{3}+1\right)y = -1+8\sqrt{3}$$

In this case, the equation of the other line is $\left(\sqrt{3}-2\right)x + \left(2\sqrt{3}+1\right)y = -1+8\sqrt{3}$.

Case II : $m_2 = \dfrac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$

The equation of the line passing through point (2, 3) and having a slope of $\dfrac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$ is

$$(y-3) = \frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}(x-2)$$
$$\left(2\sqrt{3}-1\right)y - 3\left(2\sqrt{3}-1\right) = -\left(2+\sqrt{3}\right)x + 2\left(2+\sqrt{3}\right)$$
$$\left(2\sqrt{3}-1\right)y + \left(2+\sqrt{3}\right)x = 4+2\sqrt{3}+6\sqrt{3}-3$$
$$\left(2+\sqrt{3}\right)x + \left(2\sqrt{3}-1\right)y = 1+8\sqrt{3}$$

In this case, the equation of the other line is $\left(2+\sqrt{3}\right)x + \left(2\sqrt{3}-1\right)y = 1+8\sqrt{3}$.

Thus, the required equation of the other line is $\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$ or $\left(2+\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$.

Question 13:

Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).

The right bisector of a line segment bisects the line segment at 90°.

The end-points of the line segment are given as A (3, 4) and B (–1, 2).

Accordingly, mid-point of AB $=\left(\frac{3-1}{2},\frac{4+2}{2}\right)=(1,3)$

Slope of AB $=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$

∴Slope of the line perpendicular to AB = $-\frac{1}{\left(\frac{1}{2}\right)}=-2$

The equation of the line passing through (1, 3) and having a slope of –2 is

$(y-3)=-2(x-1)$

$y-3=-2x+2$

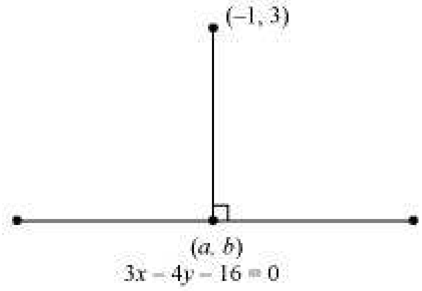
$2x+y=5$

Thus, the required equation of the line is $2x+y=5$.

Question 14:

Find the coordinates of the foot of perpendicular from the point (–1, 3) to the line $3x-4y-16=0$.

Let (a, b) be the coordinates of the foot of the perpendicular from the point $(-1, 3)$ to the line $3x - 4y - 16 = 0$.



Slope of the line joining $(-1, 3)$ and (a, b), $m_1 = \frac{b-3}{a+1}$

Slope of the line $3x - 4y - 16 = 0$ or $y = \frac{3}{4}x - 4,\ m_2 = \frac{3}{4}$

Since these two lines are perpendicular, $m_1 m_2 = -1$

$$\therefore \left(\frac{b-3}{a+1}\right) \times \left(\frac{3}{4}\right) = -1$$
$$\Rightarrow \frac{3b-9}{4a+4} = -1$$
$$\Rightarrow 3b - 9 = -4a - 4$$
$$\Rightarrow 4a + 3b = 5 \quad \ldots(1)$$

Point (a, b) lies on line $3x - 4y = 16$.

$\therefore 3a - 4b = 16$ … (2)

On solving equations (1) and (2), we obtain

$$a = \frac{68}{25} \text{ and } b = -\frac{49}{25}$$

Thus, the required coordinates of the foot of the perpendicular are $\left(\frac{68}{25}, -\frac{49}{25}\right)$.

Question 15:

The perpendicular from the origin to the line $y = mx + c$ meets it at the point

(–1, 2). Find the values of m and c.

The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at (–1, 2).

Therefore, the line joining the points (0, 0) and (–1, 2) is perpendicular to the given line.

∴Slope of the line joining (0, 0) and (–1, 2) $= \frac{2}{-1} = -2$

The slope of the given line is m.

$$\therefore m \times -2 = -1 \qquad [\text{The two lines are perpendicular}]$$
$$\Rightarrow m = \frac{1}{2}$$

Since point (–1, 2) lies on the given line, it satisfies the equation $y = mx + c$.

$$\therefore 2 = m(-1) + c$$
$$\Rightarrow 2 = \frac{1}{2}(-1) + c$$
$$\Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$.

Question 16:

If p and q are the lengths of perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \operatorname{cosec} \theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

The equations of given lines are

$x \cos \theta - y \sin\theta = k \cos 2\theta$ … (1)

$x \sec\theta + y \operatorname{cosec}\theta = k$ … (2)

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

On comparing equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \cos\theta$, $B = -\sin\theta$, and $C = -k \cos 2\theta$.

It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$\therefore p = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k \cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = |-k \cos 2\theta| \qquad ...(3)$$

On comparing equation (2) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \sec\theta$, $B = \operatorname{cosec}\theta$, and $C = -k$.

It is given that q is the length of the perpendicular from (0, 0) to line (2).

$$\therefore q = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}} \qquad ...(4)$$

From (3) and (4), we have

$$p^2+4q^2=\left(\left|-k\cos 2\theta\right|\right)^2+4\left(\frac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}\right)^2$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\sec^2\theta+\operatorname{cosec}^2\theta\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+4k^2\sin^2\theta\cos^2\theta$$

$$=k^2\cos^2 2\theta+k^2\left(2\sin\theta\cos\theta\right)^2$$

$$=k^2\cos^2 2\theta+k^2\sin^2 2\theta$$

$$=k^2\left(\cos^2 2\theta+\sin^2 2\theta\right)$$

$$=k^2$$

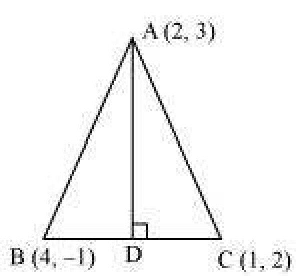
Hence, we proved that $p^2 + 4q^2 = k^2$.

Question 17:

In the triangle ABC with vertices A (2, 3), B (4, –1) and C (1, 2), find the equation and length of altitude from the vertex A.

Let AD be the altitude of triangle ABC from vertex A.

Accordingly, AD⊥BC



The equation of the line passing through point (2, 3) and having a slope of 1 is

$(y-3) = 1(x-2)$

$\Rightarrow x - y + 1 = 0$

$\Rightarrow y - x = 1$

Therefore, equation of the altitude from vertex A = $y - x = 1$.

Length of AD = Length of the perpendicular from A (2, 3) to BC

The equation of BC is

$$(y+1) = \frac{2+1}{1-4}(x-4)$$
$$\Rightarrow (y+1) = -1(x-4)$$
$$\Rightarrow y+1 = -x+4$$
$$\Rightarrow x+y-3=0 \qquad ...(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$.

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain A = 1, B= 1, and C = –3.

$$\therefore \text{Length of AD} = \frac{|1\times2+1\times3-3|}{\sqrt{1^2+1^2}} \text{ units} = \frac{|2|}{\sqrt{2}} \text{ units} = \frac{2}{\sqrt{2}} \text{ units} = \sqrt{2} \text{ units}$$

Thus, the equation and the length of the altitude from vertex A are $y - x = 1$ and $\sqrt{2}$ units respectively.

Question 18:

If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and$b$, then show that $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$.

It is known that the equation of a line whose intercepts on the axes are *a* and *b* is

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\text{or } bx+ay=ab$$
$$\text{or } bx+ay-ab=0 \qquad \ldots(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by
$$d=\frac{\left|Ax_1+By_1+C\right|}{\sqrt{A^2+B^2}}.$$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = b$, $B= a$, and $C = -ab$.

Therefore, if p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we obtain

$$p=\frac{\left|A(0)+B(0)-ab\right|}{\sqrt{b^2+a^2}}$$
$$\Rightarrow p=\frac{\left|-ab\right|}{\sqrt{a^2+b^2}}$$

On squaring both sides, we obtain

$$p^2=\frac{(-ab)^2}{a^2+b^2}$$
$$\Rightarrow p^2\left(a^2+b^2\right)=a^2b^2$$
$$\Rightarrow \frac{a^2+b^2}{a^2b^2}=\frac{1}{p^2}$$
$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence, we showed that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.